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Vibrations of Skew Cantilever Plates

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REFERENCES 1 and 2 describe the results of calculations of frequencies and nodal lines of vibrating rectangular cantilever plates. These calculations have been extended to vibrating skew cantilever plates. An IBM Fortran program is available. The plate is assumed to be vibrating transversely, in a single harmonic. Figure 1 gives the geometry of the plate.

In Refs. 1 and 2, the solution was obtained by a Fourier-series method. Here, it was found convenient to use the Rayleigh-Ritz method. The mathematical development is the same as that of Ref. 3. The result of applying the Rayleigh-Ritz method is an infinite, real, symmetric matrix, for which the eigenvalues and eigenvectors are to be calculated. In the calculations the matrix is truncated in the usual fashion to successive finite-order matrices, and the limits of the eigenvalues and eigenvectors are evaluated numerically. The method used for calculating the eigenvalues and eigenvectors of the finite matrices is developed in Ref. 4.

In order to obtain any degree of accuracy in calculating the eigenvectors, and therefore the nodal lines, it has been found absolutely essential that the same finite-order matrix be used as was used for the eigenvalues. In fact, it is desirable to calculate the eigenvalues to several more significant figures than the number required for the nodal lines. This is in direct contrast to the Fourier-series method described in Refs. 1 and 2, where an estimated limit for the frequencies was used in calculating the nodal lines.

References 1 and 2 describe the variations of the frequencies and the nodal lines as functions of the ratio of sides a/b for a rectangular plate. Thus, the different harmonics were thought of as "frequency curves." It is now possible

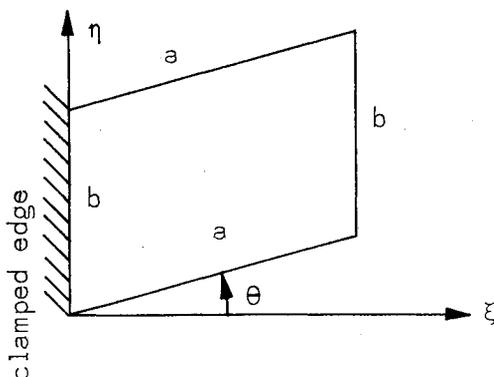


Fig. 1 Geometry of the plate

to consider the frequencies and nodal lines as functions of two independent variables, a/b and θ . Instead of referring to "frequency curves," it is now possible to refer to "frequency sheets."

References 1 and 2 referred to "transition points," points at which the frequency curves should have crossed each other but actually refused to do so, markedly changing their curvature instead. It is now possible to state the existence of "transition curves," curves along which the frequency sheets refuse to cross each other but instead markedly change their curvature. However, along different segments of a transition curve a wide variation is possible in the distance between two frequency sheets. In fact, the sheets can actually touch (become tangent to) each other at isolated points.

As in Refs. 1 and 2, the nodal lines rotate about one or several points as the frequency sheets change their curvature. Reference 5 gives a detailed description of the mathematical development and the results, as well as the Fortran program used in calculating the eigenvalues and eigenvectors.

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An Example of Boundary Layer Formation

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A MAJOR difficulty in the teaching of fluid dynamics is the lack of a simple exact solution of the Navier-Stokes equations in which both the viscous and inertial forces are active. Viscous forces only are involved in Poiseuille and Couette flows, and consequently the velocity fields are independent of the Reynolds number. There are two exact solutions that depend on viscous and inertial forces, namely the von Kármán flow produced by a rotating disk and the Jeffrey-Hamel flow in a converging or diverging channel. Interesting as these solutions are, they suffer from the disadvantage of requiring the solution of nonlinear differential equations, and the velocity fields cannot be expressed in simple terms. For teaching purposes, a solution is required which can be expressed in simple functions, is exact, and involves a balance between viscous and inertial forces, so that the dependence on the Reynolds number can be exhibited and the formation of a boundary layer as the Reynolds number increases demonstrated. It also would be helpful if a class of solutions rather than a single solution were known, to provide examples for the student to find for himself. It does not seem to be recognized widely that a class of solutions satisfying these requirements exists, a particular case

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